Dualities in dense baryonic (quark) matter with chiral and isospin imbalance

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Group

small group

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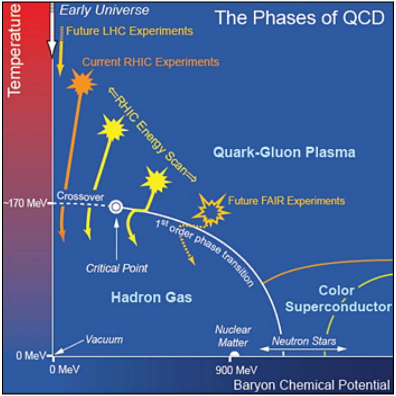
QCD at finite temperature and nonzero chemical potential

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in many different physical systems. (QCD at extreme conditions)

- neutron stars
- heavy ion collision experiments
- Early Universe



Methods of dealing with QCD

Methods of dealing with QCD

- First principle calcaltion – lattice Monte Carlo simulations, LQCD
- Effective models

  Nambu–Jona-Lasinio model NJL

lattice QCD at non-zero baryon chemical potential μ_B

Lattice QCD

non-zero baryon chemical potential μ_B

sign problem — complex determinant

$$(Det(D(\mu)))^{\dagger} = Det(D(-\mu^{\dagger}))$$

(3+1)-dimensional NJL model

NJL model can be considered as **effective field theory for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

Parameters G, Λ, m_0

chiral limit $m_0 = 0$

in many cases chiral limit is a very good approximation

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

Relative simplicity allow to consider hot and dense QCD in the framework of NJL model and explore the QCD phase structure (diagram).

chiral symmetry breaking

Unlike the QED , **the QCD vacuum has non-trivial structure** due to non-perturbative interactions among quarks and gluons

GOR relation and lattice simulations $\Rightarrow$ **condensation of quark and anti-quark pairs**

$$\langle \bar{q}q \rangle \neq 0, \quad \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \approx (-250 MeV)^3$$

Nambu–Jona-Lasinio model

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\right]$$

$$q \to e^{i\gamma_5\alpha} q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\left[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\right]q - \frac{N_c}{4G}\left[\sigma^2 + \pi^2\right].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\left[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\right]q$$

Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

The corresponding term in the Lagrangian is

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \text{ where } \mu \text{ -quark chemical potential}$$

Isotopic chemical potential μ_I

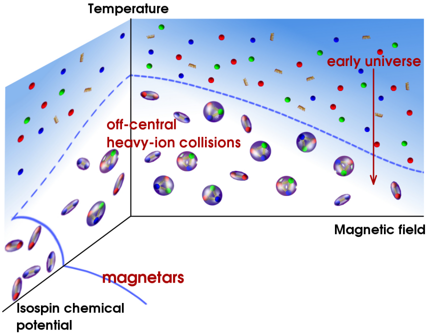
Allow to consider systems with isotopic imbalance.

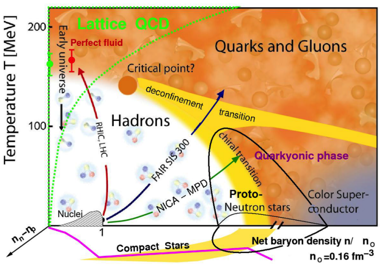
$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

The corresponding term in the Lagrangian is $\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q$

QCD phase diagram with isotopic imbalance

neutron stars, heavy ion collisions have isotopic imbalance





Different types of chemical potentials: chiral imbalance

chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between between densities of left-handed and right-handed quarks).

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

Different types of chemical potentials: chiral imbalance

chiral (axial) isotopic chemical potential

Allow to consider systems with chiral isospin imbalance

$$\mu_{I5} = \mu_{u5} - \mu_{d5}$$

so the corresponding density is

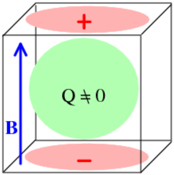
$$n_{I5} = n_{u5} - n_{d5}$$

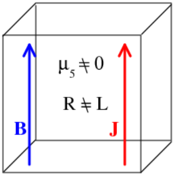
$$n_{I5} \quad \longleftrightarrow \quad \mu_{I5}$$

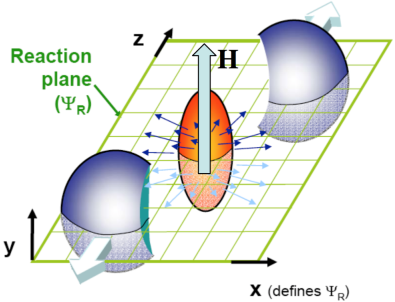
Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q$

If one has all four chemical potential, one can consider different densities n_{uL}, n_{dL}, n_{uR} and n_{dR}

Chiral magnetic effect





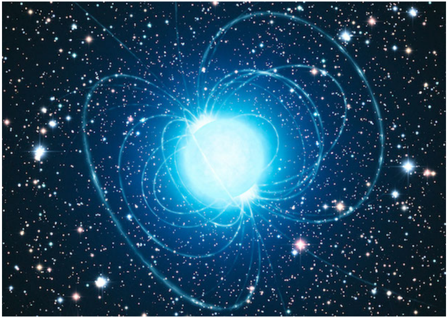


$$\vec{J} = c\mu_5 \vec{B}, \qquad c = \frac{e^2}{2\pi^2}$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033 [arXiv:0808.3382 [hep-ph]].

Chiral separation effect

Chiral imbalance could appear in compact stars



$$\vec{J}_5 = c\mu\vec{B}, \qquad c = \frac{e^2}{2\pi^2}$$

there is current and there is n_5

Model and its Lagrangian

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k $(k = 1, 2, 3)$ are Pauli matrices.

quark masses, chiral limit

light quarks u, d

$$m_u = 0.005 \text{ GeV}, \quad m_d = 0.009 \,\text{GeV}$$

chiral limit $m_u = m_d = 0$

Equivalent Lagrangian

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^1 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big].$$

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates x,

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0. \qquad (1)$$

where M and Δ are already constant quantities.

thermodynamic potential

the thermodynamic potential can be obtained in the large N_c limit

$$\Omega(M, \Delta)$$

Projections of the TDP on the M and Δ axes

No mixed phase $(M \neq 0, \Delta \neq 0)$

it is enough to study **the projections** of the TDP on the M and Δ

projection of the TDP on the M axis $F_1(M) \equiv \Omega(M, \Delta = 0)$

projection of the TDP on the Δ axis $F_2(\Delta) \equiv \Omega(M = 0, \Delta)$

Dualities

The TDP (phase daigram) is invariant

Interchange of condensates

matter content

$$\Omega(C_1, C_2, \mu_1, \mu_2)$$

$$\Omega(C_1, C_2, \mu_1, \mu_2) = \Omega(C_2, C_1, \mu_2, \mu_1)$$

Dualities of the TDP

The TDP is invariant with respect to the so-called duality transformations (dualities)

1) The main duality

$$\mathcal{D}: \ M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

$$\nu \longleftrightarrow \nu_5 \text{ and PC} \longleftrightarrow \text{CSB}$$

2) Duality in the CSB phenomenon

$$F_1(M) \text{ is invariant under } \mathcal{D}_M \ : \ \nu_5 \leftrightarrow \mu_5$$

3) Duality in the PC phenomenon

$$F_2(\Delta) \text{ is invariant under } \mathcal{D}_\Delta \ : \ \nu \leftrightarrow \mu_5$$

PC phenomenon breaks $\mathcal{D}_M$ and CSB phenomenon $\mathcal{D}_\Delta$ duality

Dualities in different approaches

- Similar dualities between chiral and superconducting condensates in low dimensional models due to Pauli Gursey, D. Ebert, T.G. Khunjua, K.G. Klimenko, V.Ch. Zhukovsky, Phys. Rev. D 90, 045021 (2014), Phys. Rev. D 93, 105022 (2016)

- **Large N_c orbifold equivalences** connect gauge theories with different gauge groups and **matter content** in the large N_c limit.
  M. Hanada and N. Yamamoto,
  JHEP 1202 (2012) 138, arXiv:1103.5480 [hep-ph],
  PoS LATTICE **2011** (2011), arXiv:1111.3391 [hep-lat]
  two gauge theories with G_1 and G_2 with μ_1 and μ_2
  Duality $G_1 \longleftrightarrow G_2$, $\mu_1 \longleftrightarrow \mu_2$
  G_1 is sign problem free
  G_2 has sign problem, can not be studied on lattice

Dualities in large N_c orbifold equivalences

two gauge theories with gauge groups G_1 and G_2 with μ_1 and μ_2

Duality

$$G_1 \longleftrightarrow G_2, \; \mu_1 \longleftrightarrow \mu_2$$

G_2 is sign problem free

G_1 has sign problem, can not be studied on lattice

Dualities in large N_c limit of NJL model

$$\Omega(C_1, C_2, \mu_1, \mu_2)$$

Duality

$$C_1 \longleftrightarrow C_2,$$
$$\mu_1 \longleftrightarrow \mu_2$$

QCD with μ_1 —- sign problem free,
and with μ_2 has sign problem, can not be studied on lattice

Pion condensation history

Phase structure of the (3+1) dim NJL model

In the early 1970s Migdal suggested the possibility of pion condensation in a nuclear medium A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2210 (1971) [Sov. Phys. JETP 36, 1052 (1973)]; A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rept. 192, 179 (1990). R.F. Sawyer, Phys. Rev. Lett. 29, 382 (1972);

From the results of the experiments concerning the repulsive πN interaction pion condensation is highly unlikely to be realized in nature A. Ohnishi D. Jido T. Sekihara, and K. Tsubakihara .

Very brief history and motivation

There has been a lot of activity in this area
pion condensation in NJL_4
K. G. Klimenko, D. Ebert J.Phys. G32 (2006) 599-608
arXiv:hep-ph/0507007
K. G. Klimenko, D. Ebert
Eur.Phys.J.C46:771-776,(2006) arXiv:hep-ph/0510222

also in **(1+1)- dimensional case**, NJL_2
K. G. Klimenko, D. Ebert, PhysRevD.80.125013 arXiv:0902.1861 [hep-ph]

$$\Downarrow$$

pion condensation in dense matter **predicted without certainty**

physical quark mass – no pion condensation in dense medium
H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri
Phys.Rev.D79:034032,2009 arXiv:0809.2658 [hep-ph]

Phase structure of (3+1)-dim NJL model

Phase structure of the (3+1) dim NJL model

Chiral isospin chemical potential μ_{I5} generates charged pion condensation in the dense quark matter.

(ν, ν_5) phase portrait of NJL_4

Duality between chiral symmetry breaking and pion condensation

$$\mathcal{D}: \quad M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

$$\mathrm{PC} \longleftrightarrow \mathrm{CSB} \quad \nu \longleftrightarrow \nu_5$$

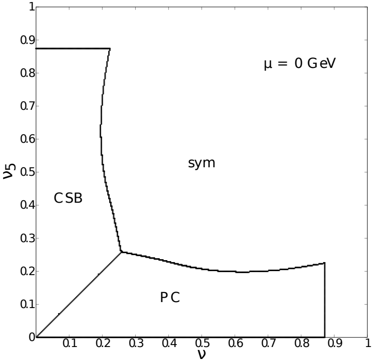


Figure: (ν, ν_5) at $\mu = 0$ GeV

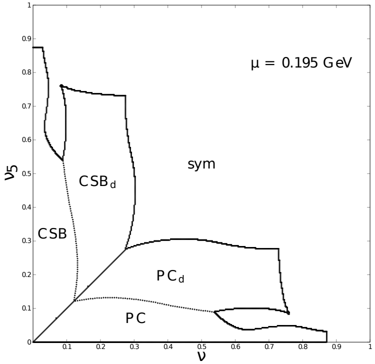


Figure: (ν, ν_5) at $\mu = 0.195$ GeV

Consideration of the case with μ_B, μ_I, μ_{I5} and μ_5 chemical potentials in (3+1)-dimensional NJL model

$$(\mu_B,\, \mu_I,\, \mu_{I5},\, \mu_5),$$
$$(\nu_5 = \frac{\mu_{I5}}{2},\, \nu = \frac{\mu_I}{2})$$

Up to now $(\mu_B,\, \mu_I,\, \mu_{I5})$ was considered $(\mu_{I5} \neq 0$ and $\mu_5 = 0)$

Now let us consider μ_5 instead of μ_{I5} $(\mu_5 \neq 0,\, \mu_{I5} = 0)$

$$(\mu_B,\, \mu_I,\, \mu_{I5}) \longrightarrow \boldsymbol{(\mu_B,\, \mu_I,\, \mu_5)}$$

How **chiral imbalance** in the form of **chiral μ_5 chemical potential** influence PC condensation

Chiral imbalance in the form of μ_5 chemical potential. (ν, μ_5) phase diagram

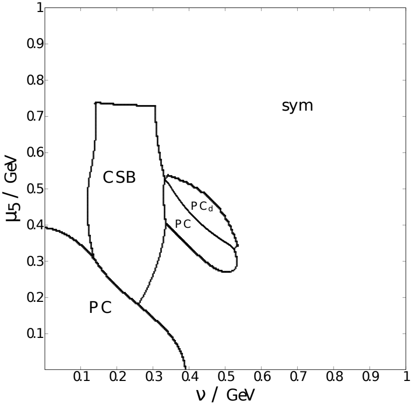


Figure: (ν, μ_5) phase diagram at $\mu = 0.23$ GeV.

Chiral chemical potential μ_5 generates charged pion condensation in the dense quark matter as well.

$$\mu_5 \rightarrow \text{PC}_\text{d}$$

-Not so prominently as μ_{I5} does

-But only at comparatively low densities n_q

Duality in the PC phenomenon

Duality in the PC phenomenon

PC phenomenon ($F_2(\Delta)$) is invariant under $\mathcal{D}_\Delta$

$$\mathcal{D}_\Delta \ : \ \nu \leftrightarrow \mu_5$$

But CSB does not respect the duality $\mathcal{D}_\Delta$ so one has to check that CSB is dynamically suppressed in the duality conjugated regions

CSB is dynamically suppressed $M_0 = 0$

Consideration of the general case μ, μ_I, μ_{I5} and μ_5

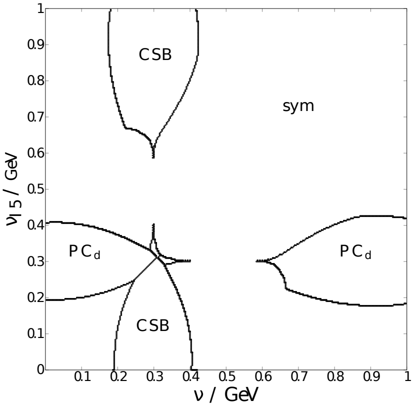


Figure: (ν, ν_5) phase diagram at $\mu_5 = 0.5$ GeV and $\mu = 0.3$ GeV.

In this case the phase diagram even richer

generation of PC_d phase is even more widespread

Comparison with lattice QCD

Comparison with lattice QCD, finite temperate and physical point

- Before that point we considered the **chiral limit**

$$m_0 = m_u = m_d = 0$$

$$m_0 \neq 0, \quad m_0 \approx 5 \text{ MeV}$$

- For that let us consider the finite temperature T

duality is approximate

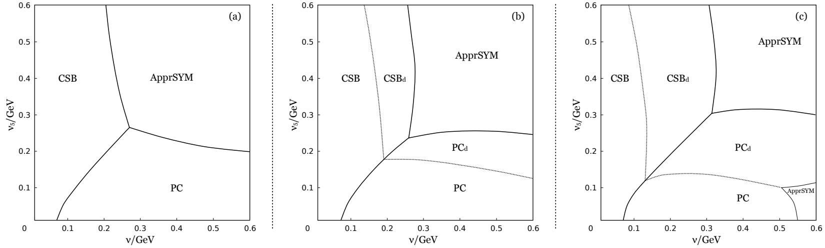
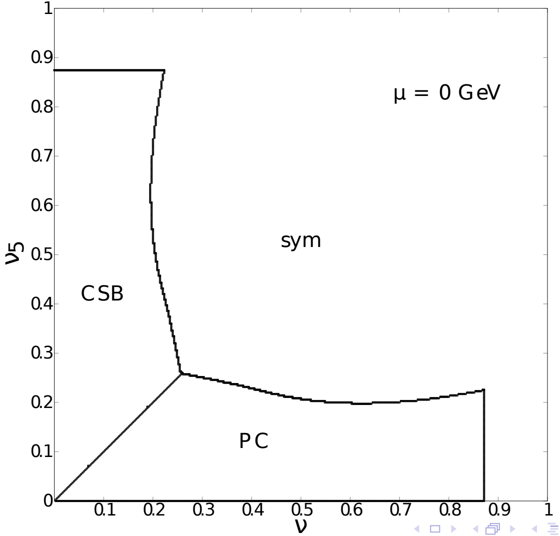


Figure: (ν, ν_5) phase diagram

(ν, ν_5) phase portrait of NJL_4 at $\mu = 0$

The case of $\mu = 0$ can be considered on lattice



Comparison with lattice QCD

Comparison with lattice QCD

The case of zero baryon chemical potential μ can be considered on lattice

Two cases have been considered in LQCD

– QCD at non-zero isospin chemical potential μ_I has been considered in arXiv:1611.06758 [hep-lat],
Phys. Rev. D 97, 054514 (2018) arXiv:1712.08190 [hep-lat]
Endrodi, Brandt et al (see B. Brandt talk)

– QCD at non-zero chiral chemical potential μ_5 has been considered in Phys. Rev. D 93, 034509 (2016) arXiv:1512.05873 [hep-lat]
Braguta, ITEP lattice group (see V. Braguta talk)

QCD at non-zero isospin chemical potential μ_I: (ν, T) phase portrait comparison between NJL model and lattice QCD

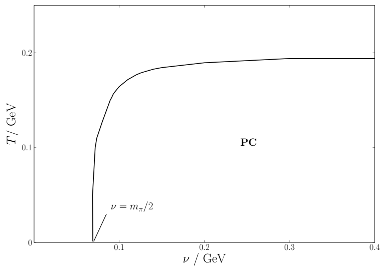


Figure: (ν, T) phase diagram at $\mu = 0$ and $\nu_5 = 0$ GeV

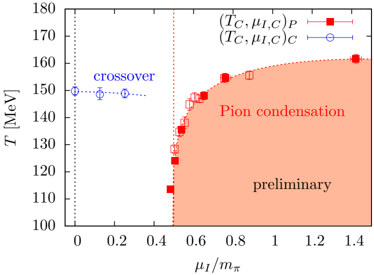


Figure: (ν, T) phase diagram arXiv:1611.06758 [hep-lat]

QCD at non-zero isospin chemical potential μ_I: (ν, T) phase portrait comparison between NJL model and lattice QCD

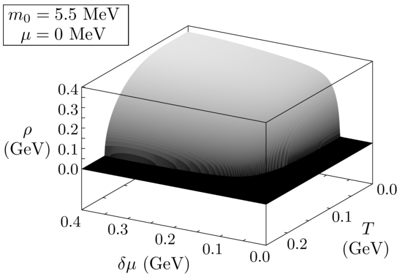


Figure: (ν, T) phase diagram at $\nu_5 = 0$ GeV from J. Phys. G: Nucl. Part. Phys. 37 015003 (2010)

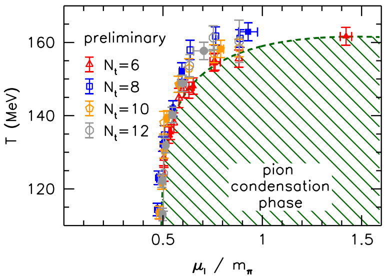


Figure: (ν, T) phase diagram at $\nu_5 = 0$ GeV arXiv:1611.06758 [hep-lat]

ChPT has similar phase diagram

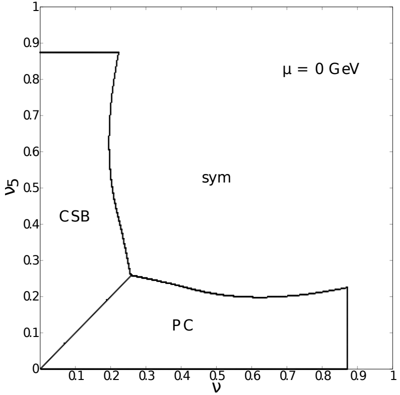
D.T. Son, M.A. Stephanov Phys.Rev.Lett. 86 (2001) 592-595
arXiv:hep-ph/0005225
Phys.Atom.Nucl.64:834-842,2001; Yad.Fiz.64:899-907,2001
arXiv:hep-ph/0011365

QCD with non-zero chiral chemical potential μ_5

QCD at zero baryon chemical potential $\mu = 0$
but with non-zero $\mu_5 \neq 0$ sign problem free

$\mu_5 \neq 0$ **no sign problem**

Braguta ITEP lattice, Ilgenfritz Dubna et al
SU(2), SU(3) – Catalysis of Dynamical Chiral Symmetry Breaking



μ_5 or ν_5 chemical potential, duality

CSB phenomenon is invariant under

$$\mathcal{D}_M \; : \; \nu_5 \leftrightarrow \mu_5$$

(μ_5, T) and (ν_5, T) are the same

QCD at non-zero chiral chemical potential μ_5, comparison between NJL model and lattice QCD

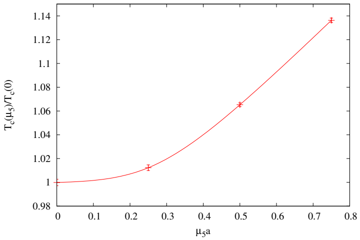


Figure: critical temperature T_c as a function of μ_5 in LQCD, from arXiv:1512.05873 [hep-lat

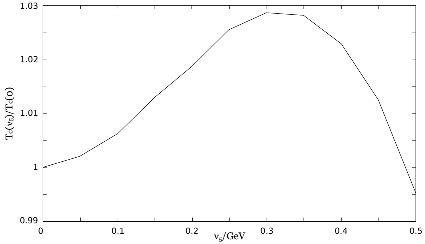


Figure: critical temperature T_c as a function of μ_5 in the framework of NJL model

Charge neutrality condition

the general case $(\mu,\ \mu_I,\ \mu_{I5},\ \mu_5)$

consider charge neutrality case $\quad \rightarrow \quad \nu = \mu_I/2 = \nu(\mu, \nu_5, \mu_5)$

Charge neutrality condition

-pion condensation in dense matter predicted without certainty,
at ν there is a small region of PC_d phase
K. G. Klimenko, D. Ebert J.Phys. G32 (2006) 599-608
arXiv:hep-ph/0507007

-physical quark mass and electric neutrality - no pion condensation in dense medium
H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri
Phys.Rev.D79:034032,2009 arXiv:0809.2658 [hep-ph]

-Chiral isospin chemical potential μ_{I5} generates PC_d

-can this generation happen in the case of neutrality condition

Charge neutrality condition

It can be shown that the PC_d phase can be generated by chiral imbalance in the case of charge neutrality condition
25pt

non-zero $\mu_5 \rightarrow PC_d$ phase in neutral quark matter

(1+1)-dimensional Gross-Neveu (GN) or NJL model consideration

(1+1)- dimensional GN, NJL model

(1+1)-dimensional Gross-Neveu (GN) or NJL model possesses a lot of common features with QCD

- renormalizability
- asymptotic freedom
- sponteneous chiral symmetry breaking in vacuum
- dimensional transmutation
- have the similar $\mu_B - T$ phase diagrams

NJL_2 model
laboratory for the qualitative simulation of specific properties of QCD at arbitrary energies

Phase structure of (1+1)-dim NJL model

Phase structure of the (1+1) dim NJL model

Chiral isospin chemical potential μ_{I5} generates charged pion condensation in the dense quark matter.

Phys. Rev. D 95, 105010 (2017) arXiv:1704.01477 [hep-ph]
Phys. Rev. D 94, 116016 (2016) arXiv:1608.07688 [hep-ph]

Phase portrait (μ, ν, ν_5) of NJL_2

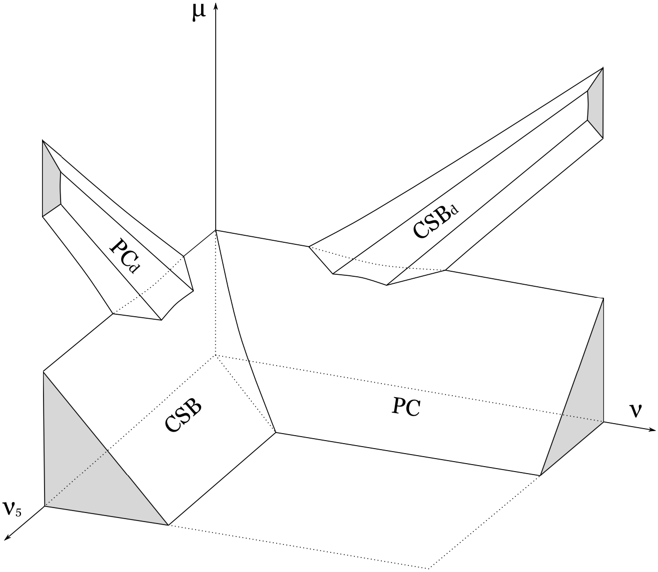


Figure: (μ, ν, ν_5) phase diagram in homogeneous case

Comparison of phase diagram of (3+1)-dim and (1+1)-dim NJL models

Comparison of phase diagram of (3+1)-dim and (1+1)-dim NJL models

The phase diagrams obtained in two models that are assumed to describe QCD phase diagram are qualitatively the same

(μ, ν) phase portraits comparison, NJL_2 and NJL_4

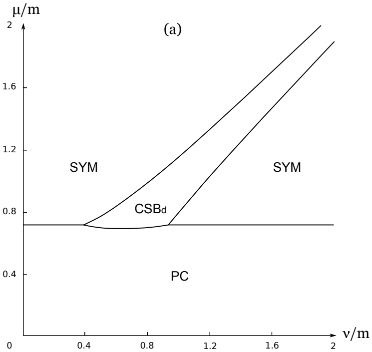


Figure: (μ, ν) phase diagram in the framework of NJL_2 model at $\nu_5 = 0$ GeV

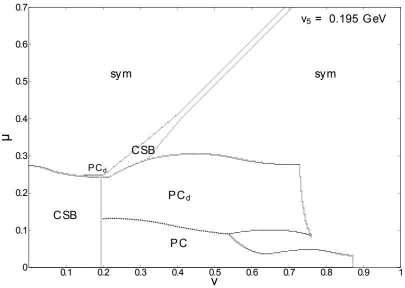


Figure: (μ, ν) phase diagram in the framework of NJL_4 model at $\nu_5 = 0.195$ GeV

Conclusions

$\mu_B \neq 0$ - dense quark matter
$\mu_I \neq 0$ isotopically asymmetric
$\mu_5 \neq 0$ and $\mu_{I5} \neq 0$ chirally asymmetric

CSB and PC in NJL model

Dualities; duality between CSB and PC: $\nu_5 \leftrightarrow \nu$

$\mu_{I5} \rightarrow \mathrm{PC_d}$

Both μ_{I5}, μ_5: **wide PC_d generation** even with **neutrality condition**

Thanks for the attention

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